

Initial state radiation effects in inclusive J/ψ production at B factories

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ABSTRACT: Based on the Monte Carlo techniques, we analyze the initial state radiation effects in prompt J/ψ inclusive production at B factories. The initial state radiation will enhance the cross section $\sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ by about 15–25%, which is almost the same size with QCD correction and relativistic correction. On the other hand, it only changes $\sigma(e^-e^+ \rightarrow J/\psi + c\bar{c} + X)$ slightly. The J/ψ momentum spectrum both in $e^-e^+ \rightarrow J/\psi + gg + X$ and in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ will be softer after photon showering from initial e^\pm beams radiation. After combining QCD, relativistic and initial state radiation corrections, we get more precise theoretical results. Especially, the new result will give a more stringent constraint of the color-octet contribution to $\sigma(e^-e^+ \rightarrow J/\psi + X_{\text{non-}c\bar{c}})$.

KEYWORDS: [NRQCD](#), [ISR](#), [Charm](#).

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1. Introduction

A good way of studying the interplay between perturbative QCD and non-perturbative QCD is investigating the quarkonium production at various colliders. Prompt J/ψ production in hadronic collisions indicates that color-octet mechanism should play a crucial role in reducing the discrepancies between theory and experiments [1, 2, 3, 4, 5, 6, 7]. Although color-singlet contribution may receive large QCD corrections in several examples [8, 9, 10, 11, 12, 13], it encounters the difficulty in explaining the yields and polarisation of prompt J/ψ simultaneously in hadronic collisions. In other words, a significant color octet component is essential to understand the large transverse momentum hadronic data in the current framework. On the contrast, in a relative small scale physics regime, it seems that color singlet one is already sufficient to explain the experimental data both at hadron colliders [14, 15] and at B factories [16, 17, 18, 19, 20, 21, 22]. Hence, it is necessary to study the physics at these two scales further.

In this paper, we will consider the prompt J/ψ inclusive production in e^-e^+ collisions at center-of-mass energy $\sqrt{s} = 10.6$ GeV. A decade ago, **BABAR** collaboration and **BELLE** collaboration reported the cross section $\sigma(e^-e^+ \rightarrow J/\psi + X)$ to be $2.52 \pm 0.21 \pm 0.21$ pb [23] and $1.47 \pm 0.10 \pm 0.13$ pb [24] respectively. In constrast, the leading-order (LO) color-singlet theoretical prediction [25, 26, 27, 28, 29, 30], which mainly includes contributions from the processes $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ and $e^-e^+ \rightarrow J/\psi + gg + X$, was at least 3 – 5 times lower than the measurements. This fact would suggest there might be substantial color-octet contribution, which can be generated at lower α_s power via $e^-e^+ \rightarrow J/\psi(^3P_J^{[8]}, ^1S_0^{[8]}) + g$. Later, **BELLE** collaboration measured the associated production cross section $\sigma(e^-e^+ \rightarrow J/\psi + c\bar{c} + X) = 0.87^{+0.21}_{-0.19} \pm 0.17$ pb and the ratio $R_{c\bar{c}} = \frac{\sigma(e^-e^+ \rightarrow J/\psi + c\bar{c} + X)}{\sigma(e^-e^+ \rightarrow J/\psi + X)} = 0.59^{+0.15}_{-0.13} \pm 0.12$ [31].

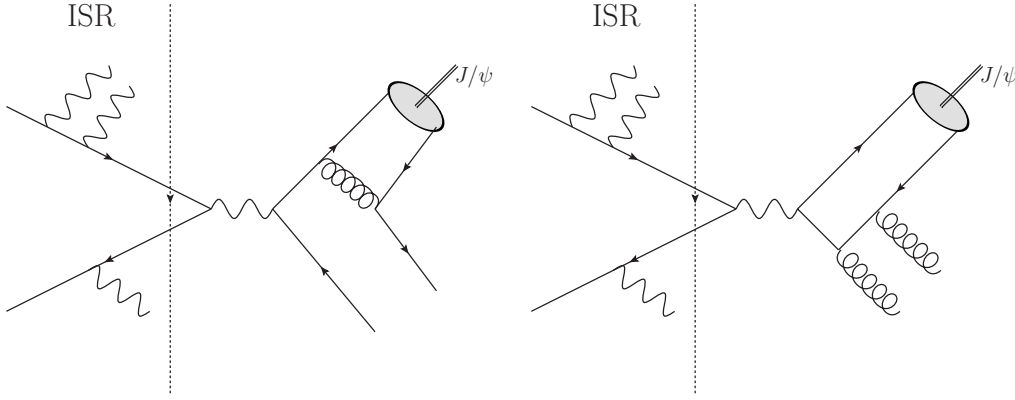


Figure 1: Two representative Feynman diagrams with ISR.

The cross section for $J/\psi + c\bar{c} + X$ is at least a factor of 5 larger than LO color-singlet [26, 27, 28, 29, 30, 32] and color-octet [32] theoretical estimations. Several theoretical improvements were made afterward to reduce the large discrepancies [33, 34, 35]. Especially, next-to-leading order (NLO) QCD corrections to gain a large enhancement [16] of the cross section for $J/\psi + c\bar{c} + X$, which was also confirmed by other authors [17]. The result can be comparable with the experimental data at least with some specific parameter choices. Along the same line, both QCD correction [18, 19] and relativistic correction [20, 21] are able to enhance the color-singlet cross section $\sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ by about 20 – 30%. Moreover, QCD correction [18, 19] to $e^-e^+ \rightarrow J/\psi + gg + X$ will significantly change the J/ψ momentum distribution, especially near the kinematic endpoint. It is important since color-octet part [36] will enhance the cross section at the endpoint before performing resummation [33]¹. In other words, the J/ψ momentum spectrum might give a constraint to color-octet matrix elements. A comparison of theoretical result with the up-to-date **BELLE** measurements [37] was performed in Ref.[22]. A strong constraint on the non-perturbative color-octet matrix elements was extracted as [22]

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle + 4.0 \frac{\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle}{m_c^2} < (2.0 \pm 0.6) \times 10^{-2} \text{ GeV}^3, \quad (1.1)$$

which is apparently in contradiction with the large p_T hadronic results in non-relativistic QCD (NRQCD) theory [38] at NLO level [1, 2, 3, 4, 5, 6, 7].

We will study the initial state radiation (ISR) effect to prompt J/ψ inclusive production at B factories in the article. Initial state radiation is a very important

¹It is heavily based on a phenomenological shape function [33].

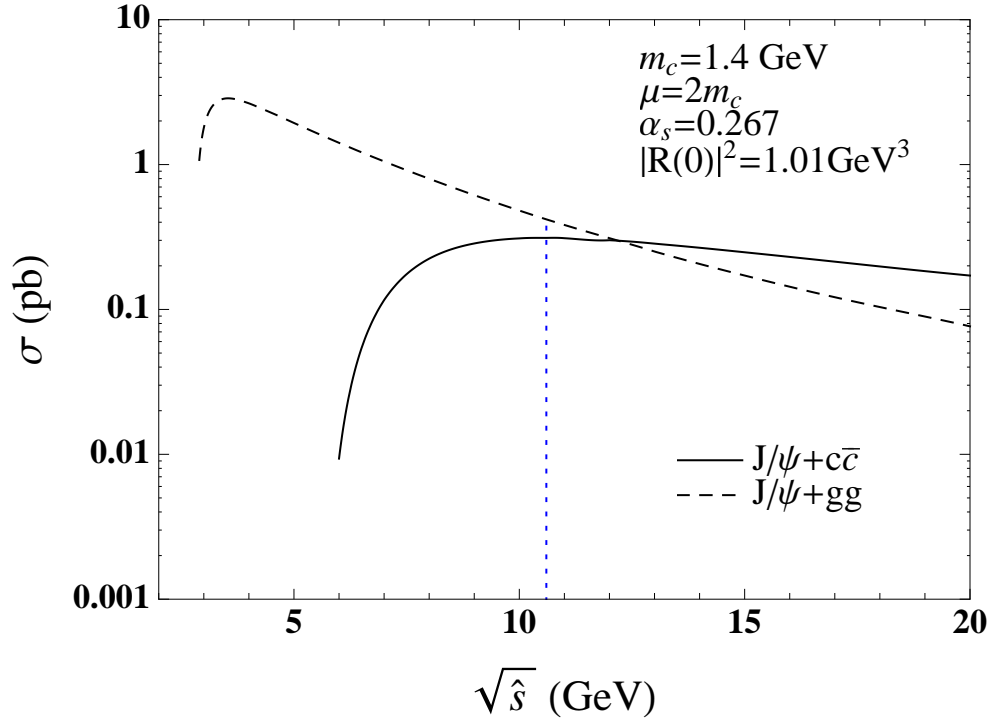


Figure 2: Cross sections as functions of center-of-mass energy $\sqrt{\hat{s}}$ in hard reaction.

ingredient that should be understood in investigating physics in electron-positron collisions. After including ISR effect in inclusive J/ψ production at B factories, we will obtain a more precise theoretical result. In general, the detailed studies of photon radiative corrections from initial e^\pm beams require Monte Carlo generators [39, 40, 41, 42, 43, 44, 45, 46, 47]. We interfaced a general-purposed matrix element and events generator **HELAC-Onia** [48, 49, 50, 51, 52] to the general photon shower program **QEDPS** [45, 46, 47] to include the initial state radiation in various e^-e^+ annihilation processes. In the above two interested processes $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ and $e^-e^+ \rightarrow J/\psi + gg + X$, two representative Feynman diagrams with ISR are shown in Fig.1. After photon shower, the annihilating e^- and e^+ no more make the head-on collision, as they might deviate from the beam axis by the radiation. The center-of-mass energy after showering $\sqrt{\hat{s}}$ will be smaller than $\sqrt{s} = 10.6$ GeV. The LO cross section for $e^-e^+ \rightarrow J/\psi + gg + X$, as shown in Fig.2, increases as $\sqrt{\hat{s}}$ decreases near 10.6 GeV, while that for $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ changes very little as $\sqrt{\hat{s}}$ decreases near 10.6 GeV. Therefore, we would expect that ISR effect should be significant in $J/\psi + gg + X$, while it might be not so important in $J/\psi + c\bar{c} + X$. Moreover, since the factorizable of ISR and J/ψ production, ISR should not change QCD correction and relativistic correction unless the K factors of these two corrections are varying a lot with $\sqrt{\hat{s}}$ near 10.6 GeV. Next, we will study the ISR effect in detail by including QCD correction and relativistic correction.

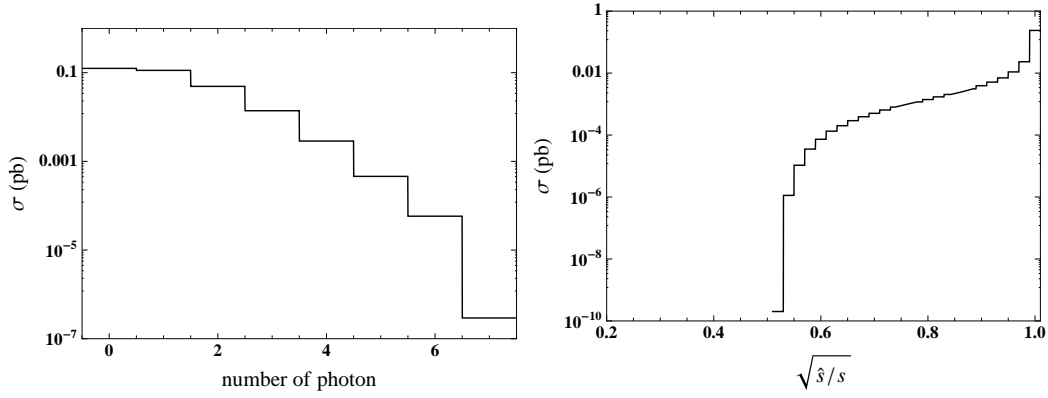


Figure 3: Cross section as functions of number of photon and $\sqrt{\hat{s}}/\sqrt{s}$ in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$.

The organization of the following article is: we will study the ISR effects in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ in section 2 and in $e^-e^+ \rightarrow J/\psi + gg + X$ in section 3, and draw our conclusion in section 4.

2. ISR in $J/\psi c\bar{c} + X$

The inclusive double charm production at B factories is one of the most interesting processes to probe the heavy quarkonium physics. The most precise measurement of its cross section by **BELLE** collaboration is [37]

$$\sigma_{\text{prompt}}(e^-e^+ \rightarrow J/\psi + c\bar{c} + X) = 0.74 \pm 0.08^{+0.09}_{-0.08} \text{ pb.} \quad (2.1)$$

Color-singlet cross section with NLO QCD correction is 0.33(0.47) pb when $m_c = 1.5(1.4)$ GeV and $\mu = 2m_c, |R(0)|^2 = 1.01 \text{ GeV}^3$. The QED and double photon contributions will enhance the cross section by 8+29 fb [22]. The feeddown contribution from $\psi(2S)$ will enlarge the cross section by a factor of 1.355, while that from χ_{cJ} is 21 fb [22, 32]. The small color-octet contribution is 11 fb [32]. After combining all of these contributions, the prompt cross section will become 0.51(0.71) pb [22]. The relativistic correction was done in Ref.[53], and the authors found it was negligible.

However, the physical cross section should always include ISR at e^-e^+ annihilation. In Fig.3, we show the cross sections with number of radiated photon for $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$. There are substantial probability to radiate at least a photon. The averaged number of ISR photon in each event is about 0.88, which can be imagined since though there is a α suppression for radiating a photon, there is an extra $\log(m_e/\sqrt{s})$ enhancement. In order to have a look at how the center-of-mass energy

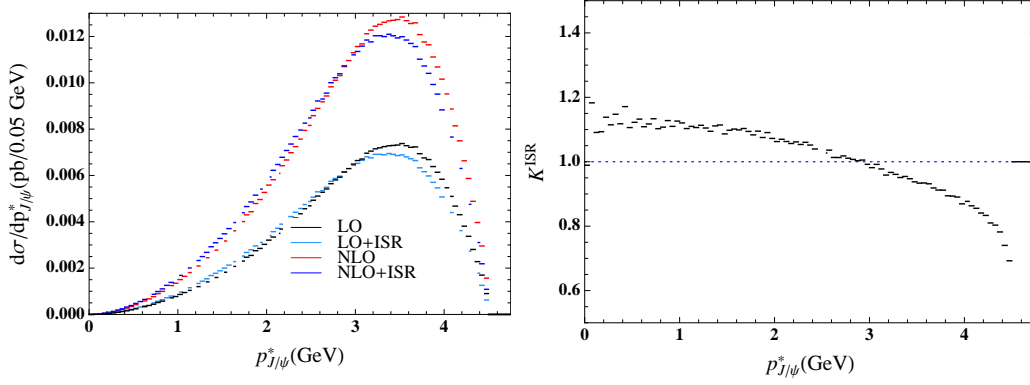


Figure 4: Cross sections and $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ as functions of J/ψ momentum $p_{J/\psi}^*$ in rest frame of initial e^-e^+ . We take the parameter set as $m_c = 1.4$ GeV, $\mu = 2m_c$ in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$.

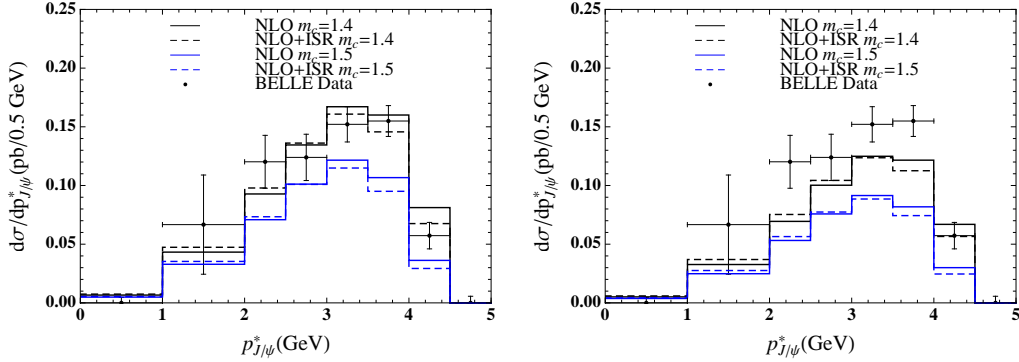


Figure 5: Comparisons of the theoretical predictions and **BELLE** measurement [37] with $\mu = 2m_c$ (left-panel) and $\mu = \sqrt{s}/2$ (right-panel) respectively in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$. We have multiplied a factor 1.355 to account in the feeddown contribution from $\psi(2S) \rightarrow J/\psi + X$.

$\sqrt{\hat{s}}$ changes after showering, we also plot the $\sqrt{\hat{s}}/\sqrt{s}$ distribution in Fig.3. The averaged value for $\sqrt{\hat{s}}/\sqrt{s}$ is about 0.98. It is quite close to 1, and it also indicates ISR correction should be small.

Fig.4 shows the ISR effect in J/ψ momentum spectrum. We obtained the curves for NLO and NLO + ISR by normalizing the corresponding LO and LO + ISR² results by a NLO K factor from Ref.[22], because the K factor changes mildly in $p_{J/\psi}^*$ ³

²Here, LO + ISR means LO with ISR effects. LO result has already including QCD and QED diagrams.

³Here, $p_{J/\psi}^*$ means the momentum of J/ψ in the rest frame of initial e^-e^+ before showering.

parameter sets	LO (pb)	LO+ISR (pb)	NLO (pb)	NLO+ISR (pb)
$m_c = 1.4 \text{ GeV}, \mu = 2m_c$	0.45	0.44	0.77	0.75
$m_c = 1.5 \text{ GeV}, \mu = 2m_c$	0.31	0.30	0.54	0.53
$m_c = 1.4 \text{ GeV}, \mu = \sqrt{\hat{s}}/2$	0.31	0.31	0.59	0.59
$m_c = 1.5 \text{ GeV}, \mu = \sqrt{\hat{s}}/2$	0.23	0.22	0.42	0.42

Table 1: Cross sections of $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ in different parameter sets.

spectrum and with $\sqrt{\hat{s}}$ [17]. Here, we take $m_c = 1.4 \text{ GeV}, \mu = 2m_c, |R(0)|^2 = 1.01 \text{ GeV}^3$. As expected, ISR correctes the momentum spectrum only a little. To make things clear, we also present the $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ as a function of $p_{J/\psi}^*$ in the right panel of Fig.4. ISR makes the J/ψ momentum spectrum a little softer. We also compared the **BELLE** measurement [37] with the theoretical prompt results in Fig.5. Because of the uncertainties in input parameters like m_c, μ , we compared the experimental result with the theoretical ones in different parameter sets. It is shown that $m_c = 1.4 \text{ GeV}, \mu = 2m_c$ is the closest set to the **BELLE** data [37], though there are still large uncertainties in experimental data. Finally, the prompt total cross sections are summarized in Tab.1. ISR decreases the cross section by a very little amount.

3. ISR in $J/\psi gg + X$

In this section, we will study $e^-e^+ \rightarrow J/\psi + gg + X$. **BELLE** collaboration has measured the cross section for $J/\psi + X_{\text{non-}c\bar{c}}$ in Ref.[37] as

$$\sigma_{\text{prompt}}(e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}) = 0.43 \pm 0.09 \pm 0.09 \text{ pb.} \quad (3.1)$$

On the theoretical side, the NLO color-singlet cross section for prompt $J/\psi + gg + X$ is $0.67(0.53) \text{ pb}$ when $m_c = 1.4 \text{ GeV}, \mu = 2.8(5.3) \text{ GeV}, |R(0)|^2 = 1.01 \text{ GeV}^3$ [18, 19], which enhances LO cross section by about 20 – 30%. Later, it was found in Ref.[20] that the relativistic correction also contribute a factor of 20 – 30% to the color-singlet $\sigma(e^+e^- \rightarrow J/\psi + gg + X)$, which is comparable to that arises from QCD correction. Relativistic correction was also confirmed by other author [21]. From their calculations, the color-singlet result is already saturating the experimental data if we think the whole partonic cross section contributes to $J/\psi + X_{\text{non-}c\bar{c}}$ final states.

As we discussed in the first section, ISR should also change the cross section significantly. We plotted the number of ISRphoton distribution and $\sqrt{\hat{s}}/\sqrt{s}$ in Fig.6. Compared with $J/\psi + c\bar{c} + X$, there is a little larger probability to radiate a photon

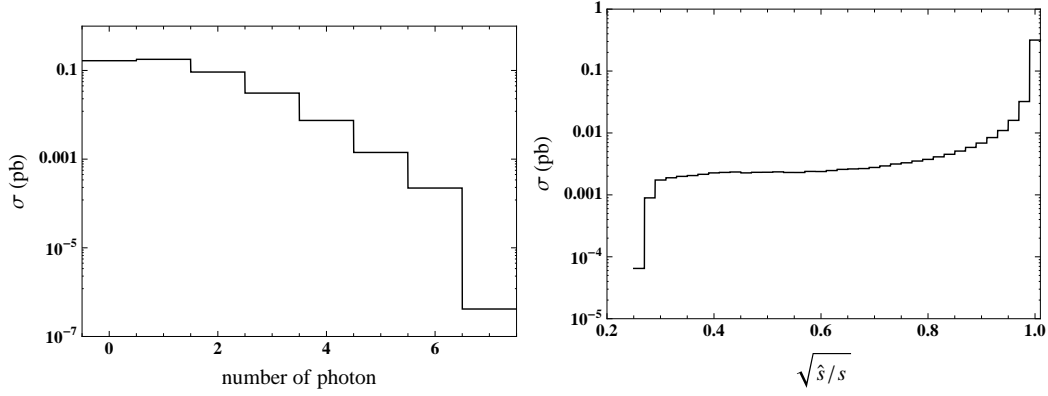


Figure 6: Cross section as functions of number of photon and $\sqrt{\hat{s}}/\sqrt{s}$ in $e^-e^+ \rightarrow J/\psi + gg + X$.

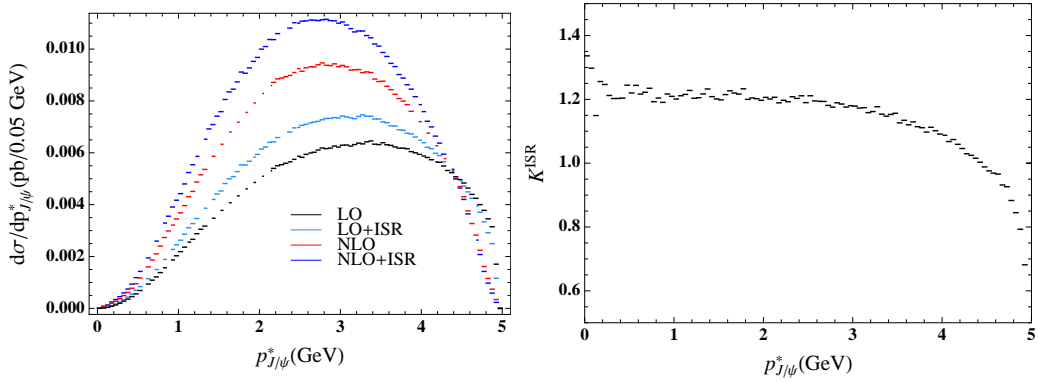


Figure 7: Cross sections and $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ as functions of J/ψ momentum $p_{J/\psi}^*$ in rest frame of initial e^-e^+ . We take the parameter set as $m_c = 1.4 \text{ GeV}, \mu = 2m_c$ in $e^-e^+ \rightarrow J/\psi + gg + X$.

in $J/\psi + gg + X$. The reason is mainly relying in the fact that the cross section $\sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ will increase as $\sqrt{\hat{s}}$ becomes smaller. The averaged number of photon in per event is enlarging to 1.04. Meanwhile, the averaged $\sqrt{\hat{s}}/\sqrt{s}$ is 0.93. In other words, ISR effect is much more important in $e^-e^+ \rightarrow J/\psi + gg + X$.

Next, we are intending to include QCD correction, relativistic correction and ISR in our color-singlet results. Unlike the case in $J/\psi + c\bar{c}$, the QCD correction to $J/\psi + gg$ will make the J/ψ momentum spectrum much softer than the LO one [18], since at the endpoint, the LO result will suffer from large logarithms $\log(1 - E_{J/\psi}/E_{J/\psi}^{\text{max}})$ due to kinematic reasons. The LO spectrum will change significantly at the endpoint with resummation of such logarithms, while resummation affects NLO spectrum in

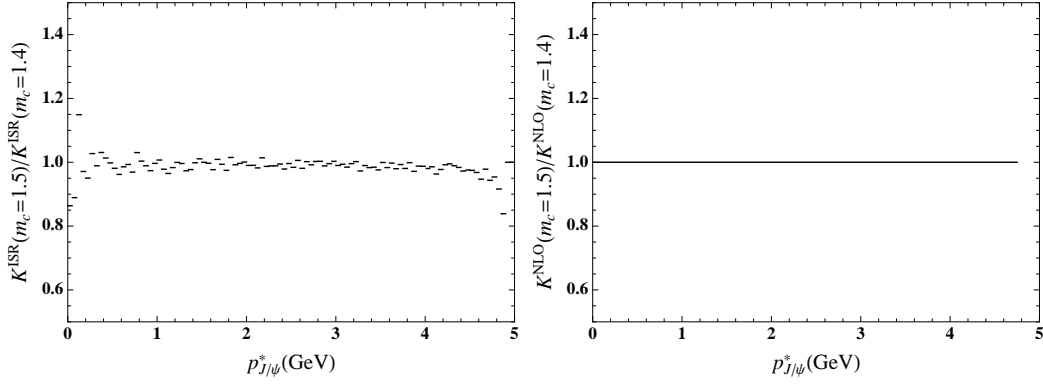


Figure 8: $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ (left) and $K^{\text{NLO}} = \sigma^{\text{NLO}}/\sigma^{\text{LO}}$ (right) as functions of J/ψ momentum $p_{J/\psi}^*$ with $m_c = 1.5$ GeV and $m_c = 1.4$ GeV and $\mu = 2m_c$ in $e^-e^+ \rightarrow J/\psi + gg + X$ than in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$.

a very limited amount [18]⁴. On the other hand, the relativistic correction should not change the LO spectrum but only enhance it by a simple K factor. We use the formula

$$\begin{aligned} \frac{d\sigma^{\text{NLO}}}{dp_{J/\psi}^*} &= (K^{\text{NLO}(\alpha_s)} + K^{\text{NLO}(v^2)} - 1) \frac{d\sigma^{\text{LO}}}{dp_{J/\psi}^*} \\ K^{\text{NLO}(\alpha_s)} &= \frac{d\sigma^{\text{NLO}(\alpha_s)}}{dp_{J/\psi}^*} / \frac{d\sigma^{\text{LO}}}{dp_{J/\psi}^*}, \\ K^{\text{NLO}(v^2)} &= \frac{d\sigma^{\text{NLO}(v^2)}}{dp_{J/\psi}^*} / \frac{d\sigma^{\text{LO}}}{dp_{J/\psi}^*}, \end{aligned} \quad (3.2)$$

to get the fixed-order result by taking account QCD and relativistic corrections in. Similar formula can be applied to the result with ISR

$$\frac{d\sigma^{\text{NLO+ISR}}}{dp_{J/\psi}^*} = (K^{\text{NLO}(\alpha_s)} + K^{\text{NLO}(v^2)} - 1) \frac{d\sigma^{\text{LO+ISR}}}{dp_{J/\psi}^*}. \quad (3.3)$$

It is justified by the fact that ISR and QCD/relativistic correction can be factorized out. Moreover, the K factor of QCD/relativistic correction changes mildly with $\sqrt{\hat{s}}$ [19, 20]. The result is shown in Fig.7. ISR makes the J/ψ momentum spectrum softer, which is much clear from $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ that is shown in the right panel of Fig.7. Another interesting thing is we want to see whether the K factors are sensitive to m_c values. We established two plots in Fig.8. From it, we see both of $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ and $K^{\text{NLO}} = K^{\text{NLO}(\alpha_s)} + K^{\text{NLO}(v^2)} - 1$ are insensitive to m_c .

⁴Resummation has only a very small effect on the total cross section as well [18].

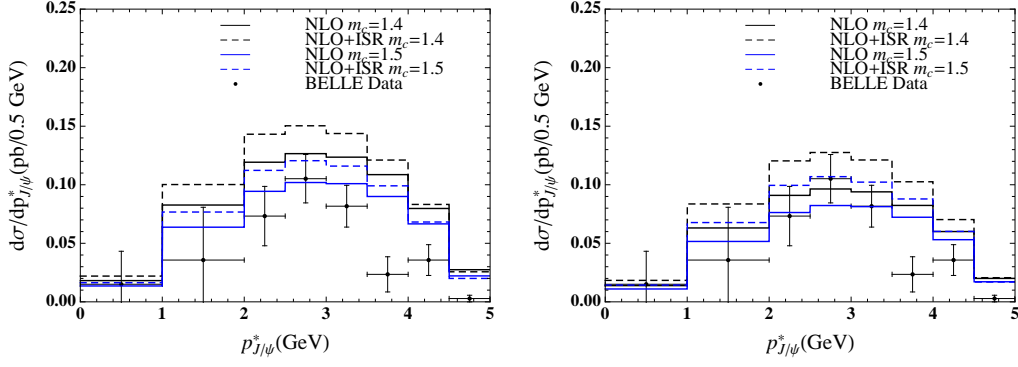


Figure 9: Comparisons of the theoretical predictions and **BELLE** measurement [37] with $\mu = 2m_c$ (left-panel) and $\mu = \sqrt{s}/2$ (right-panel) respectively in $e^-e^+ \rightarrow J/\psi + gg + X$. We have multiplied a factor 1.355 to account in the feeddown contribution from $\psi(2S) \rightarrow J/\psi + X$.

parameter sets	LO (pb)	LO+ISR (pb)	NLO (pb)	NLO+ISR (pb)
$m_c = 1.4 \text{ GeV}, \mu = 2m_c$	0.57	0.65	0.79	0.91
$m_c = 1.5 \text{ GeV}, \mu = 2m_c$	0.45	0.50	0.63	0.72
$m_c = 1.4 \text{ GeV}, \mu = \sqrt{s}/2$	0.35	0.45	0.60	0.77
$m_c = 1.5 \text{ GeV}, \mu = \sqrt{s}/2$	0.30	0.37	0.51	0.64

Table 2: Cross sections of $e^-e^+ \rightarrow J/\psi + gg + X$ in different parameter sets.

parameter sets	LO	LO+ISR	NLO	NLO+ISR	BELLE
$m_c = 1.4 \text{ GeV}, \mu = 2m_c$	0.44	0.41	0.49	0.45	0.63 ± 0.11
$m_c = 1.5 \text{ GeV}, \mu = 2m_c$	0.41	0.38	0.46	0.42	0.63 ± 0.11
$m_c = 1.4 \text{ GeV}, \mu = \sqrt{s}/2$	0.47	0.41	0.50	0.44	0.63 ± 0.11
$m_c = 1.5 \text{ GeV}, \mu = \sqrt{s}/2$	0.43	0.38	0.45	0.40	0.63 ± 0.11

Table 3: Comparisons of $R_{c\bar{c}}$ in different parameter sets with **BELLE** measurement [37].

To compare with **BELLE** measurement, we take the same bin size as theirs. The J/ψ momentum spectrum is shown in Fig.9. We take four different input parameter sets. The color-singlet result is already saturating the experiment data. With all of the three corrections (i.e. QCD correction, relativistic correction and ISR correction), there is more stringent room left for color-octet contribution in $J/\psi + X_{\text{non-}c\bar{c}}$. The total theoretical cross sections for $e^-e^+ \rightarrow J/\psi + gg + X$ in various parameter sets are summarized in Tab.2. ISR enlarges the cross section about 15 – 25%. Although the cross sections are a little bit larger than the experimental data [37], considering large theoretical uncertainties, there are still many rooms to make the theoretical result lower. For example, one can take a lower value of $|R(0)|^2$ as done in Ref.[19] or from

parameter sets	LO (pb)	LO+ISR (pb)	NLO (pb)	NLO+ISR (pb)
$m_c = 1.4 \text{ GeV}, \mu = 2m_c$	1.02	1.09	1.55	1.66
$m_c = 1.5 \text{ GeV}, \mu = 2m_c$	0.76	0.80	1.17	1.25
$m_c = 1.4 \text{ GeV}, \mu = \sqrt{s}/2$	0.66	0.76	1.19	1.36
$m_c = 1.5 \text{ GeV}, \mu = \sqrt{s}/2$	0.52	0.59	0.93	1.05

Table 4: Cross sections of $e^-e^+ \rightarrow J/\psi + X$ in different parameter sets.

potential model estimation [54]. In principle, the ratio $R_{c\bar{c}}$ should be independent of the value of $|R(0)|^2$ in color-singlet case. We presented the theoretical $R_{c\bar{c}}$ in Tab.3. We take the same parameter set in $J/\psi + c\bar{c} + X$ and $J/\psi + gg + X$ and assume $\sigma(e^-e^+ \rightarrow J/\psi + gg + X) = \sigma(e^-e^+ \rightarrow J/\psi + X_{\text{non-}c\bar{c}})$. It seems that the theoretical result is a little bit lower than **BELLE** measurement, but it is still within 2 standard deviation. Therefore, we expect that a more precise measurement will make the situation clear. Finally, we also listed the total cross sections $\sigma(e^-e^+ \rightarrow J/\psi + X) = \sigma(e^-e^+ \rightarrow J/\psi + c\bar{c} + X) + \sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ in Tab.4. It is compatible with the experiment [37] value $\sigma_{\text{prompt}}(e^-e^+ \rightarrow J/\psi + X) = 1.17 \pm 0.02 \pm 0.07 \text{ pb}$.

4. Summary

The different conclusions drawn from large p_T hadronic data and small scale e^\pm data motivate us to reconsider the theoretical results again. In this article, we keep our eyes on the issue about prompt J/ψ inclusive production at B factories. Since the cross sections in e^-e^+ annihilation usually gain large corrections from ISR. We use Monte Carlo techniques to take ISR effect into our theoretical results. We found the effect of ISR photon shower to $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ is small but it is large to $e^-e^+ \rightarrow J/\psi + gg + X$. ISR enhances the total cross section of $e^-e^+ \rightarrow J/\psi + gg + X$ by a factor of 15 – 25% depending on the input values of parameters. Moreover, ISR makes the J/ψ momentum spectrum to be softer both in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ and in $e^-e^+ \rightarrow J/\psi + gg + X$. It is important because it is thought as a good way to have a look at the color-octet contribution. Combining with QCD, relativistic and ISR corrections, we presented the theoretical results for these two processes. To compare with experiment, feeddown contributions (mainly from $\psi(2S)$) are also included. Total cross sections for $J/\psi + c\bar{c} + X$ and $J/\psi + gg + X$ are presented in Tab.1 and Tab.2 respectively, while the J/ψ momentum spectra in **BELLE** bin size [37] are shown in Fig.5 and Fig.9. Due to large experimental and theoretical uncertainties, we are still unable to draw strong conclusions, but the corrections in $J/\psi + gg + X$ really constraint the color-octet a lot. Finally, we also presented $R_{c\bar{c}}$ with assumption $\sigma(e^-e^+ \rightarrow J/\psi + gg + X) = \sigma(e^-e^+ \rightarrow J/\psi + X_{\text{non-}c\bar{c}})$, which should be more precise than the cross section alone. The theoretical result is lower

than **BELLE** measurement [37], but is still in 2σ . More careful analysis both from theoretical and experimental sides is necessary in the future.

Acknowledgments

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